Eight Stocks are Enough in China

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Abstract

This paper extends our previous work on Asia, adding a separate analysis of Chinese domestic stocks. We looked across 2,300 stocks in China (a subset of our prior universe of 13,000 stocks across Asia) over 10 years and randomly selected stocks for inclusion in equally weighted portfolios that were held for one year and then were reselected based on the new year's investable universe. In China we found that 10 stocks removed 67% of unsystematic risk. Moreover in China, adding additional stocks put heavier downward pressure on returns than in Asia. Hence we argue that to diversify risk and still maintain a good chance of outperforming the market, 8 stocks were enough in China, slightly lower than our prior finding of 10 stocks in Asia.

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I. Introduction

This study attempts to identify the optimum number of stocks that an active fund manager should hold in a portfolio of stocks in Asia (we only consider Asia excluding Japan) and a portfolio of stocks in the domestic Chinese stock market, referred to as China A shares.

The unique aspects of this study are its inclusion of all Asian markets, annual reselection, annual rebalancing, and its dynamic data set.

All markets in Asia – This research builds portfolios of companies across Asia from China H shares to shares in India.

Annual reselection – Most prior research on diversification randomly selects portfolios of two, three, four and more stocks, holds those stocks for the period (e.g., five, 10 or more years) of the study and then measures the volatility of those portfolios. The weakness of such studies is that now very few fund managers hold a portfolio for such a length of time, in fact, recent research shows that turnover of the average portfolio in the US is now reaching 100% per year. To make the results of this research more realistic, we reselect stocks annually; meaning, a portfolio of 10 stocks, reselected annually for 10 years would mean the investor owns 100 stocks over the period.

Annual rebalancing – When we reselect stocks each year we apply equal weighting to the new stocks in the portfolio.

Dynamic universe – Our list of stocks available for reselection at each year's end were actually investable at that time. The two main elements that make a stock investable are that it is large and liquid enough to which to allocate money. Each year, we remove all those which do not fit this criteria, which makes the results more robust and applicable to real

world investing. In addition, this methodology allows us to include new listings which, given the boom times in Asian markets over the past 10 years, would be a serious omission.

We start with the assumption that the active fund manager is guided by four competing objectives: to reduce uncompensated risk, to reduce complexity, to reduce costs, and to maximize return. Of these four objectives, only the first is helped by increasing the number of stocks in a portfolio; all others are better achieved by having less stocks in a portfolio. Fewer stocks means less complexity, less work and, a chance at higher than market return. The main costs come from finding that new stock, keeping track of changes happening in that company, and making decisions about when to buy more or when to sell.

Besides overcoming the performance drag of the above costs, an active fund manager is also expected to "beat the market", but the more stocks he holds, the more likely it is that his performance will mimic that of a passively managed fund.

So, the object is not only to minimize diversifiable risk, but rather to find the optimum number of stocks after which adding the next stocks fails to bring significant benefit to the investor's risk and return.

This paper starts with a literature review, follows with a description of the data-set construction, and then introduces our methodology. This is followed by our results and analysis and ends with our conclusion.

II. Literature review

One lesson the world of finance learned from Harry Markowitz's seminal work on investing (1952), commonly referred to as Modern Portfolio Theory (MPT), was that each stock's movement relative to others is unique and at times opposing, which causes risk to drop dramatically when an investor shifts focus from individual stocks to a portfolio.

William Sharpe (1964) expanded on this by developing the Capital Asset Pricing Model (CAPM), which segregated risk between those that were inherent in the stock market (market or systematic risk), and therefore cannot be removed; and those that come from the movements of individual stocks (company or unsystematic risk), which can be removed.

In 1968, Evans and Archer sparked a long line of research that showed that the most dramatic drop in portfolio volatility in the United States (US) occurs after adding only as few as eight stocks. They analyzed the effects of change in portfolio size on portfolio standard deviation. Their study on the S&P 500 stocks as of 1958 used a random selection of stocks of different portfolio sizes to measure the reduction in risk as stocks were added to a portfolio. Figure 1 generally illustrates their finding that adding about eight stocks to a randomly selected portfolio was enough to diversify away most risk and that after 19 stocks, any more stocks failed to make much difference.

A follow-on conclusion was that if company-specific risk could be nearly completely removed from a portfolio just by adding a small number of stocks, then a rational investor would take this simple action. An investor would miss out on the reward of extra return if he or she was without sufficient sense to remove this extra risk.

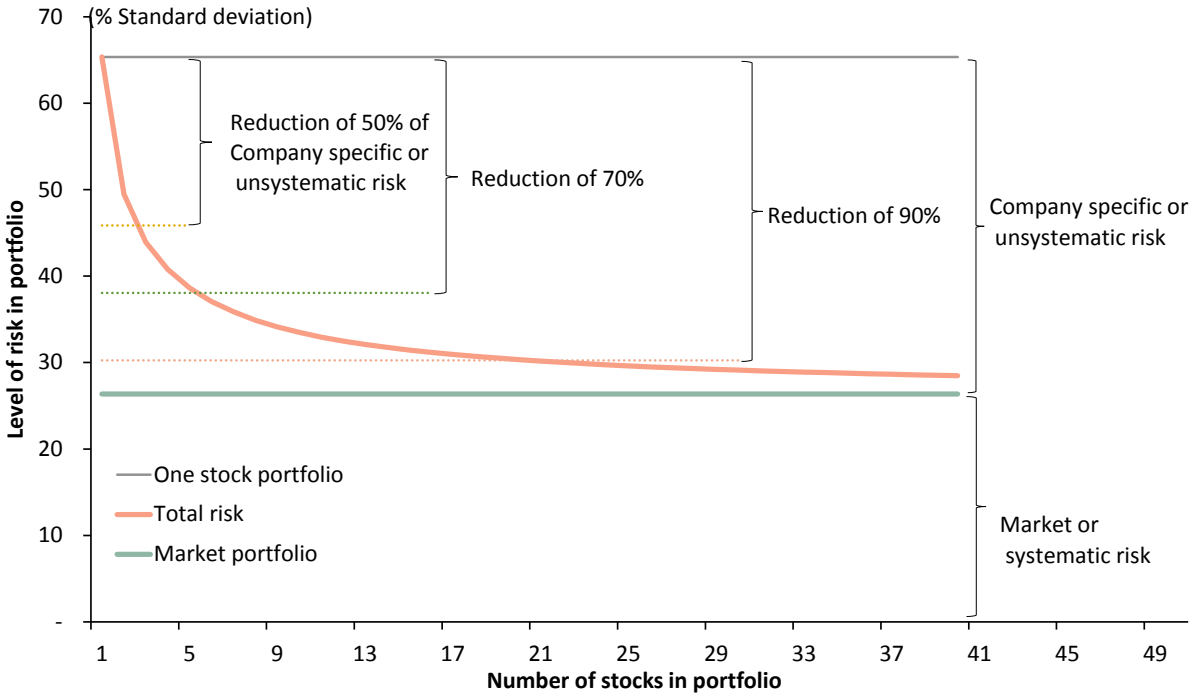

Figure 1**. Risk reduction comes fast and furious.** The separation of risk into company-specific and market risk reminds active portfolio managers to hold as few stocks in a portfolio as possible in order to improve the chance of outperforming the market portfolio.

Using an analytical expression of risk reduction relative to the number of stocks in the portfolio, Elton and Gruber (1977) put this study into practice. They then tried to reconcile the results with the outcomes from actual price data. Using US weekly returns data from 1971 to 1974, they concluded that 10 stocks were required to diversify away 75% of the total risk that would be experienced by a single-stock portfolio.

In 2005, Hyung and de Vries, studied the effect of diversification on downside risk only, in the case of both normally distributed and fat-tailed returns. They found that in a fat-tailed distribution, there is greater probability of an extreme loss or an extreme gain, thus reflecting market conditions in stressed scenarios. The study was conducted on daily returns of a randomly selected group of 15 stocks from 1980 to 2001, with the S&P 500 used as the market index. They concluded that in extreme scenarios, nearly all benefits of diversification were exhausted by holding only 10 stocks.

In 2007, Domian, Louton and Racine dealt with a hitherto unexplored area; the number of stocks required to reduce risk in terminal wealth over a longer holding period. Their research

shifted the focus from volatility over a period, to reducing the volatility at the end of the investment horizon. The measure they used to measure portfolio risk was expected shortfall. The study was done on US stocks over the 20 years from 1985 to 2005 and looked at the risk of the terminal portfolio wealth falling below a threshold of what an investor would have gained from an investment in a 20-year government bond. They concluded that 164-, 93- and 63-stock portfolios have respectively 1%, 5% and 10% chance of falling below a threshold of 9.14 times the initial wealth. Their paper questioned the prior conclusions of only eight to 20 stocks being needed to reduce most of the unsystematic risk.

Until 2010, several papers were published that claimed that randomly drawn, equally weighted portfolios performed as well as portfolios using some stock-selection method or a stock-weighting optimization model. However, Kritzman, Page and Turkington (2010) used various asset-class data in the US to show that stock-weight-optimized portfolios outperformed randomly drawn, equally weighted portfolios.

In 2010, Benjelloun revisited Evans and Archer's 1968 study to see what had changed. Using the data for all US stocks from 1980 to 2000, he analyzed time-series standard deviation and terminal-wealth standard deviation using portfolios of randomly selected stocks and then compared market weighting versus equal weighting. His work showed that, irrespective of the weighting scheme, whether for time-series standard deviation or for terminal-wealth standard deviation, after 40 to 50 stocks were in the portfolio, no further diversification benefit could be achieved by adding more stocks.

Bennet and Sias (2010) challenged the conventional wisdom that investors would achieve diversification benefits by forming portfolios containing between eight and 50 stocks. To do this, they started with all US stocks from 1999 to 2008. Then, they separated the systematic and unsystematic returns using various factor models such as a single-factor (CAPM), three-factor, and four-factor models, and all of them with an industry-specific factor. From this, the

authors showed that even with a 100-stock portfolio, the volatility of the volatility (in this case standard deviation) of unsystematic returns was still sizable at 6%, though it is negligibly higher than the standard deviation of an equally weighted market portfolio. This research refuted old conceptions that a small number of stocks was all that was needed to reduce the majority of risk.

In their recent paper, Alexeev and Tapon (2012) looked at data of common stocks in the US, United Kingdom (UK), Japan, Canada and Australia from 1975 to 2011. The authors calculated several risk measures and performance measures and, also studied the symmetric and asymmetric nature of risk. Their conclusion provided a recommended range for the number of stocks, that is, a confidence interval around the mean number of stocks, to reduce 90% of the diversifiable risk in normal periods, general market crashes and industry-specific meltdowns. The authors also stated that for longer period buy-and-hold portfolios, the number of stocks should be based on a study undertaken during normal periods, not stressed ones. The authors recommended that professional portfolio managers – using standard deviation as a measure of risk and seeking to reduce 90% of diversifiable risk, 90% of the time – should hold between 40 and 70 stocks (US); 30-65 (UK); 30-50 (Japan); 20-50 (Canada); and 30-50 (Australia).

Various research has generally reached the same conclusions – that, from a risk-reduction perspective, 10 to 20 stocks in a portfolio were all that was needed to reduce almost all uncompensated risk. It reminded us that diversification benefits come fast and furious, and therefore, that active investors should have a portfolio that is concentrated in a small number of stocks. In fact, in Stotz and Lu (2014) we identified a portfolio as low as 10 stocks in Asia as bring optimal.

What we have known since 1952 is that a relatively small portfolio is all an investor needs to reduce risk, and going beyond this would have minimal impact on the reduction of risk and

would have a negative impact on returns. This is a valuable reminder as institutional investors often hold too many stocks in their portfolios and individual investors, as reported by Goetzmann and Alok (2008), hold too few – the majority hold less than five stocks in their portfolios.

III. Data

We obtained from FactSet monthly price data for all stocks listed on every Asian stock exchange (for example, China: Shanghai and Shenzhen; India: the Bombay Stock Exchange and the National Stock Exchange) from the first day of 2003 to year end 2013.

To construct an accurate data set and to prevent survivorship bias, this list included all stocks – whether they were newly listed or delisted at any time during the period.

Table I.
All stocks that were listed over the time period
This study starts by obtaining all stocks that existed over the past 10 years in each of the major stock markets in Asia, including delistings and new IPOs over that time.

Country	Markets	All stocks
India (IN)	Bombay Stock Exchange, The National Stock Exchange of India	3,259
China (CN)	Shanghai Stock Exchange, Shenzhen Stock Exchange	2,313
Korea (KR)	Korea Stock Exchange	1,738
Taiwan (TW)	Taiwan Stock Exchange	1,666
Hong Kong (HK)	Stock Exchange of Hong Kong	1,417
Malaysia (MY)	Bursa Malaysia	896
Singapore (SG)	Stock Exchange of Singapore	694
Thailand (TH)	Stock Exchange of Thailand	520
Indonesia (ID)	Indonesia Stock Exchange	388
Philippines (PH)	Philippine Stock Exchange	218
Asia (AS)		13,109

A. Dynamic universe: Remove inactive stocks

Though we downloaded monthly data, each year we constructed portfolios using December year-end prices. Our main objective in the data-preparation phase was to make sure that the list of stocks available at each year's end was a list that was actually investable by an active fund manager at that time, a replica of the actual universe of investable stocks that a fund manager would have seen then. The two main elements that make a stock investable are that it is large (defined in terms of market capitalization) and liquid (defined in terms of average daily trading volume in the stock markets) enough to allocate money to.

From this price data we were able to calculate each stock's percentage change from the prior month for every observation. To arrive at this December year-end list of investable stocks we looked back over the prior 12 months. If we saw that there were more than six

months of no monthly price changes, we then tagged that stock "inactive" for that prior year, meaning that it was not an investable alternative for an active fund manager at the time. Hence, those "uninvestable" stocks were excluded from the investable universe at the end of that year. It was this year-end universe from which we selected the stocks to hold for the coming year.

If we excluded an "inactive" stock, it did not mean that it was excluded every year; rather it was only excluded as an investable option at that year's end. We repeated this process with all stocks at every year's end. So, if that previously "inactive" stock started to have price movement (for example, it was released from a market-imposed suspension) in the upcoming year, then it would re-enter the investable universe for that year's end. This process gave us a dynamic investable universe every year.

We performed a final test on stocks in the coming year by removing those that had price movements greater than three standard deviations from the mean. We did this as a means to clean the data and remove any possible errors or outliers. In doing this, we removed about 40 stocks from the maximum universe of 13,109. In almost all cases these companies were tiny.

The change in the universe came not only from active stocks becoming inactive, but also from inactive stocks becoming active. The main source of new stocks in the market came from newly listed stocks. In addition, some stocks fell out of the following year's universe if they were delisted or possibly acquired in an M&A deal. By creating the universe of investable stocks anew each year we were able to get very close to the reality of what an investor actually faced at that time.

This dynamic universe also allowed us to improve on past research in this area by reselecting stocks on an annual basis, rather than holding portfolios of stocks for the period of the study (a rather static and unlikely scenario in real life).

B. Dynamic universe: Remove small and illiquid stocks

Once we had our list of active stocks, we ranked each stock at each year's end based on an equally weighted composite score of its size (market capitalization) and volume (three-months-prior average daily turnover).

The top stocks were large and liquid, while the bottom stocks were small and illiquid. Our objective was to create a list of stocks from our data set of available stocks that were investable for individual and institutional investors at that point in time.

As we went through this process of removing the small and illiquid stocks for each year, we made sure that a stock that was inactive one year could still be included in another year if its size and volume increased enough.

To implement this step we moved to the country level. We started with the question: "What percentage of this market's size and liquidity would we cut off if we removed the bottom 10% of companies?" We then repeated that process for 20%, 30%, 40%, 50%, 60%, 70%, 80% and 90%. Our objective was to identify a list of truly investable stocks in each Asian market from all the stocks that were "active" in that country in that period.

To use the Philippines as an example, if we cut off the bottom the smallest and least liquid 60% of all 225 stocks, our investable universe at the end of 2012 would shrink to 85 stocks. In 2012, those 85 stocks accounted for 94% of the size of the overall market and 98% of the trading activity in the market. If we averaged these two items for 2012, we could see that, on average, these investable stocks accounted for 96% of the market's size and trading activity. If we averaged this number over the past five years, these 60% of stocks accounted for 88% of the market's size and trading activity.

Table II. Philippines investable universe selection

Philippines - Cutting off 60% of all stocks	
All stocks	225
Investable stocks	86
Excluded stocks	139
Investable stocks as a % of all stocks	38.2
Size of investable stocks as a % of all stocks	94.8
Volume of investable stocks as a % of all stocks	97.9
Average of size and volume	96.4
Five-year average of size and volume	92.9
Investable stocks: Bottom decile market cap (US$m)	236.9
Investable stocks: Bottom decile daily volume (US$m)	0.3

So, though excluding 60% of the least investable stocks in the Philippines seems a big number, we were actually removing only a small portion – only 12% of the market's size and trading activity. But, what we end up with is a true representation of the stocks that were active and tradable in that country as seen by an individual or institutional investor at that time, or truly "investable" stocks.

C. Dynamic universe: Consider minimum size and volume

After removing stocks based on percentages of size and volume, we next considered absolute numbers of market capitalization and turnover to remove any further stocks that could still be considered too small or illiquid. Since our ultimate objective was to create an investable universe for each country individually, and since each country has different characteristics, we allow for different cutoff points for each country. For each market, we next balanced our analysis of percentage of size and volume with the actual level market cap and trading volume. Our objective was to keep trading volume on average above US$200,000 per day, and market cap above US$100m.

Though large institutional funds may still find it hard to invest in these stocks, a high-net-worth individual investor could certainly buy these shares. This final check allowed us to make sure that our cutoff point actually produced a universe that was truly investable.

For the Philippines, our investable stocks accounted for just 37.8% of all stocks, which cut 5.6% off the market cap, but only 2.1% of the average daily volume of all stocks. This shows that the Philippines (along with most Asian markets) has a small number of large and liquid stocks that account for nearly all of the market's size and volume.

China, meanwhile, is a much less concentrated market with a long tail of investable, mid-sized companies. In its case, we cut off only 10% of all stocks and ended with 2,050 investable stocks. But despite leaving 90% of all stocks in the investable universe, the average of the smallest decile of stocks still had a relatively large market cap of US$243m, with a high average trading volume of US$2.7m per day. Hence, in China, we only needed to cut 4.2% of the market's size and 6.4% of the market's trading activity to create our universe.

Table III. China investable universe selection

China - Cutting off 10% of all stocks	
All stocks	2,413
Investable stocks	2,050
Excluded stocks	363
Investable stocks as a % of all stocks	85.0
Size of investable stocks as a % of all stocks	95.8
Volume of investable stocks as a % of all stocks	93.6
Average of size and volume	94.7
Five-year average of size and volume	91.1
Investable stocks: Bottom decile market cap (US$m)	243.0
Investable stocks: Bottom decile daily volume (US$m)	2.7

India is at the opposite end of the spectrum. As with China, it is also a large market with 3,474 listed stocks – but only 10% of these were investable by our definition. Even after cutting off 90% of the companies in the market, the average daily volume for the lowest decile of the 10% of stocks that remained was not even US$1m per day in 2012, coming in at US$770,000 per day. The average size of the bottom decile of the 10% of the market that was investable was large at US$526m compared to the other markets, but if we shift our cutoff to 80% of the market, this number would collapse to US$119m, too small to be included. Hence, we stuck with our 90% cutoff level.

Table IV. India investable universe selection

India - Cutting off 90% of all stocks	
All stocks	3,474
Investable stocks	330
Excluded stocks	3,144
Investable stocks as a % of all stocks	9.5
Size of investable stocks as a % of all stocks	90.2
Volume of investable stocks as a % of all stocks	83.4
Average of size and volume	86.8
Five-year average of size and volume	87.0
Investable stocks: Bottom decile market cap (US$m)	526.0
Investable stocks: Bottom decile daily volume (US$m)	0.8

Table V summarizes the outcome of our data-set construction. Some countries, such as India and Malaysia, tended to have size and volume concentrated in a small number of stocks, while China stood alone as having the lowest concentration in either size or volume. Based on this methodology, we ended with a universe of 5,318 stocks in Asia over the past 10 years from a total of 13,109 that were available during that period, or 41% of this total.

Table V. Investable stocks used in this study

Country	All stocks	Investable stocks	Investable/total (%)
India (IN)	3,259	330	10
China (CN)	2,313	2,050	89
Korea (KR)	1,738	686	39
Taiwan (TW)	1,666	657	39
Hong Kong (HK)	1,417	702	50
Malaysia (MY)	896	176	20
Singapore (SG)	694	275	40
Thailand (TH)	520	204	39
Indonesia (ID)	388	153	39
Philippines (PH)	218	85	39
Asia (AS)	13,109	5,318	41

This process allowed us to create a data set that met our criterion of being a real-world data set that replicated the actual investing options that an investor had at the end of each year of this study.

IV. Methodology

A. Calculate the average volatility of a one-stock portfolio

We start by randomly selecting one stock from our investable universe at the end of 2003. We then hold that stock for one year at which time we randomly select another stock from the universe as a replacement and hold it for the year. The initially selected stock remains in the universe so it has as equal a chance as any other to be selected again. We repeat this process each year over the 10-year period and from it we can calculate the monthly return over the period of a one-stock portfolio.

We use the same standard deviation measure as Evans and Archer (1968), Campbell et al. (2001) and Benjelloun (2010):

$$TSSD_N^i = \sqrt{\sum_{s=1}^{S} \frac{(R_S^i - \overline{R_n^i})^2}{S-1}} \quad (1)$$

Where $TSSD_N^i$ is the time-series standard deviation of an N-stock portfolio i,

$$R_S^i = \sum_{j=1}^{N} \frac{r_{j,s}^i}{N} \quad (2)$$

is the return on portfolio i at time s,

$$r_{j,s}^i$$

is the return on stock j, in portfolio i, at time s,

$$\overline{R_S^i} = \sum_{j=1}^{N} \frac{R_S^i}{S} \quad (3)$$

is the average time-series return, over time, of portfolio i,

$$\overline{TSSD_N} = \sum_{i=1}^{K} \frac{TSSD_N^i}{K} \quad (4)$$

is the average time series-standard deviation of K portfolios, each of size N

This process links various one-stock portfolios over a 10-year period and measures the volatility of the value of that portfolio regardless of what stocks are in the portfolio. We repeat this process 10,000 times (K = 10,000) and then take the average of those outcomes to

find the average return and time-series standard deviation (TSSD) that would have been experienced by the average investor holding a one-stock portfolio over that time.

B. Calculate the average volatility of a two-stock portfolio

Next we move to a two-stock portfolio, which we select randomly and weight equally. We use equal rather than market weight (or a weight based on some other factor, such as fundamentals or valuation) because:

- The average investor is unlikely to have a rigorously implemented model for weighting stocks in portfolios, and may tend toward keeping it simple by holding about an equal percentage of his or her money in each stock. Anyway, identifying a commonly used method is beyond the scope of this paper.
- For average investors, equal weighting has the advantage of forcing them to rebalance by buying more of a stock that has fallen with the proceeds of a stock that has risen.
- Prior research (Benjelloun 2010) has shown a near equivalence in outcomes between size weighting vs equal weighting.
- Nearly all the prior research papers in this area have used equal weighting, hence maintaining ours using equal weighting allows for a clearer comparison.
- Finally, the purpose of this research is not to identify some "preferred" stock selection and/or portfolio weighting methodology. Rather, it is to determine the optimum number of stocks to hold in a portfolio to reduce risk.

Next, we randomly select two stocks and combine them into a portfolio and measure the monthly average return of this portfolio. At the end of that year, we remove those stocks from that portfolio and randomly select two new stocks, equally weight them in the portfolio and hold them for the next year. Again, as long as the prior stocks held were deemed investable in this following year then they would be eligible to enter any future portfolio. We then measure

the monthly return and volatility of return and repeat this selection process each year until the end of the period.

Again, we are linking various two-stock portfolios over a 10-year period and measuring the volatility of the value of that portfolio regardless of what stocks are in the portfolio.

C. Accounting for delistings

Because we reselect stocks annually and require a six-month history of trading activity, a new listing throughout any one year will be excluded from our portfolio until the year-end annual rebalancing. A delisting during the year, however, must be properly accounted for.

Over the test period, there were nearly 1,000 delistings in Asia, which, if either excluded or not properly accounted for, could distort the results of any study on the subject. Our method of adjusting for delisting starts with a review of all delistings during the 10-year period across each market in Asia. From this we calculate the average delisting gain or loss per country, per year. We then apply this country and annual average gain or loss to any delisting that occurred in that country in that year.

For example, assume that a market rises an average of 30% in any one year. Then, say that we construct a two-stock portfolio that starts with 100 invested in each stock. If both stocks rose in line with the market, that portfolio would rise to 260 by year's end. However, now assume that instead, stock A goes up to 130 by year's end, but stock D delists in May. Based on our above methodology, we apply the average loss rate for that country in that year – let us assume it was negative 20% – we then use an exit price for stock D in May of that year of 80. We then hold this money in cash until year's end when we allocate that cash back into new stocks. This means that at year's end the portfolio would have a value of 210.

By accounting for 1,000 delistings across Asia during our test period using this method we get a more accurate representation of the results an investor really faced. The impact on our

results from this methodology can impact the volatility of a one-stock portfolio. This impact on volatility drops dramatically when two (or more) stocks are included in a portfolio.

D. Calculate average volatility of N-stock portfolio until market portfolio is reached

We continue this process for a three-, four-, and five-stock portfolio all the way to a portfolio containing all stocks in the market.

Because we have created a dynamic universe, the number of stocks in that universe changes each year, and in most cases grows due to new listings. But, to perform this research we define the market portfolio each year to consist of the number of stocks equal to the minimum universe size from any one of the 10 years. For example, Asia had a total of 5,318 stocks in its universe of investable stocks in 2012, and all had an equal chance of being chosen for the market portfolio. However, each year we used a market portfolio of 3,189 stocks as this was the minimum number of stocks available in any one year, across all of the 10 years we studied.

We call this final portfolio the market portfolio and from the movements in its value we can calculate the return and risk of the market. We refer to the TSSD of this portfolio as the "market" or "systematic" risk and consider it the risk that a passive investor would have experienced.

E. Notes on differences from prior research

E.1. Annual reselection is more realistic. The first major difference in our research compared to prior work is that we are reselecting the portfolio annually, whereas prior research may have held that portfolio for the total period under study. We believe that, given the high level of turnover in portfolios these days, this is a more realistic method. Recent research (Sapp and Yan 2009) shows that the average active fund manager sells every stock in his portfolio, to rotate into a different stock, nearly once a year.

E.2. Annual reselection allows for new listings and delistings. The second major difference is that annual reselection allows us to take into account new listings and delistings that have been massive in Asia given the boom-and-bust cycle during this period. By reselecting annually, it allows us to include these changes in our universe and therefore to create a dynamic universe that most closely matches the choices that investors were facing at that time.

E.3. Account for the impact on volatility of delistings. The third major difference in our research is that we take into account the impact of delistings during any one-year holding period. As mentioned, we adjust for delisting in each year.

F. Calculate the unsystematic risk

From our calculations of one-stock risk (total or one-stock TSSD) all the way to the risk of all stocks (market TSSD), we are arrive at the difference, which is the unsystematic risk. As formula (1) shows, both of these measures were arrived at by taking the square root of the variations. So, in order to get the correct unsystematic risk we need to take the difference of the square of total risk and market risk. We then take the square root of this number to arrive at the unsystematic risk, which in Asia is 14.7%. Of course, this risk approaches zero as stocks are added to the portfolio and ends up at zero when the market portfolio is reached.

V. Results

A. The optimum risk an active fund manager should bear

The chart below (Figure 2) shows that the average monthly TSSD of a one-stock portfolio in Asia is about 16%, while the TSSD of a portfolio holding all investable stocks is about 7%.

A passive fund manager's objective is to produce returns that match the market and to remove most unsystematic risk. On the other hand, an active fund manager's objective is to outperform the market, which his hard to do without bearing some unsystematic risk. So, an active manager is focused on the extreme left side of this figure and the passive fund manager is focused on the extreme right side.

To obtain a higher return, an active fund manager must bear a higher amount of risk. But, it does not make sense for him to put all his money in one stock and bear the maximum level of risk of 16%. This would expose him to the risk of that one company's return collapsing and also to the higher volatility of such a portfolio over time.

So, the challenge is to identify the point at which, from a risk-reduction perspective, it no longer makes sense for the active fund manager to add further stocks to a portfolio.

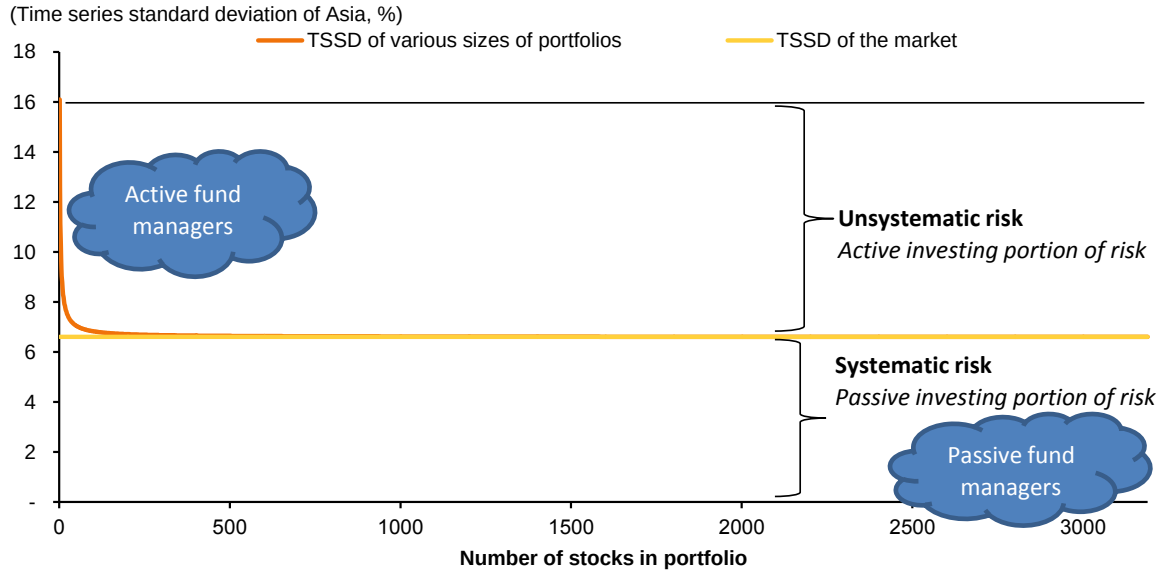

Figure 2. Segmentation of risk in Asia, from one to all stocks. This figure shows the average times-series standard deviation of 10,000 randomly drawn portfolios, for each point on the x-axis, from a one-stock portfolio to the market portfolio in Asia of 3,189 stocks.

Market risk in Asia over the period of this study was 6.6%, while China's market risk was nearly double that at 10.5%. Single-stock risk was 16.1% in Asia and a slightly lower 15.2% in China. The big difference in China compared to Asia was that the average correlation coefficient of returns between stocks in Asia was a low 0.164, which is understandable since stocks were being chosen from 10 different markets, with each market driven by very different factors. The Chinese stocks in our study had an average correlation coefficient over the period of a much higher 0.455. To put it simply, during the period of our study, Chinese stocks moved in the same direction much more than in other countries.

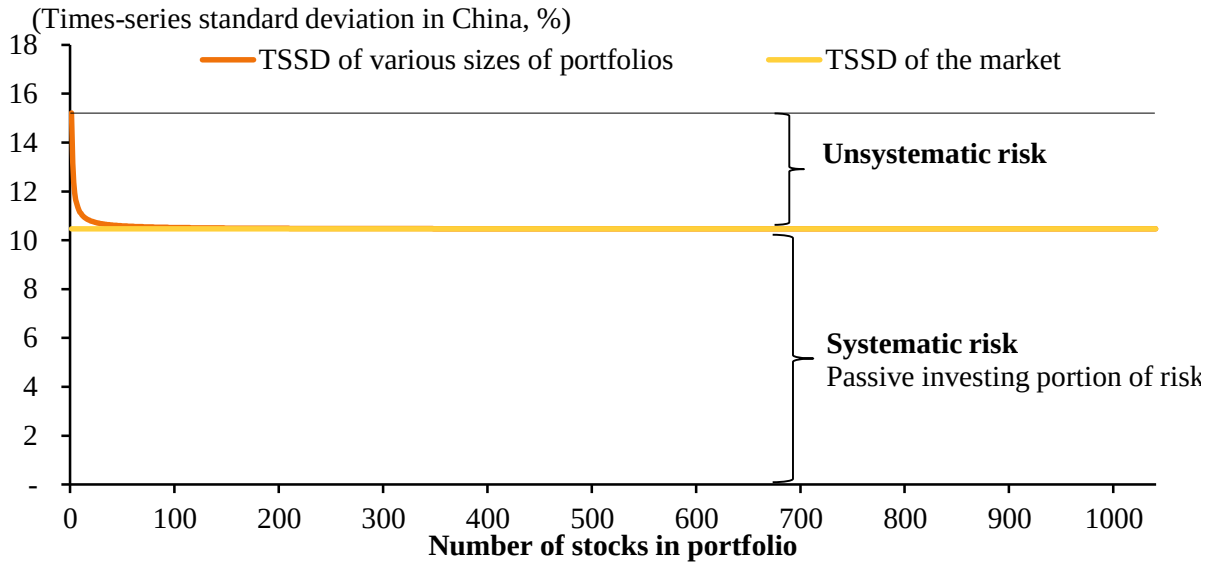

Figure 3. Segmentation of risk in China, from one to all stocks. This figure shows the average times-series standard deviation of 10,000 randomly drawn portfolios, for each point on the x-axis. from a one-stock portfolio to the market portfolio in China of 1,040 stocks.

Figure 4 shortens the x-axis to an only 50-stock portfolio, rather than the above 3,189-stock portfolio for Asia. It shows that, due to its exponential decay, there is significant benefit gained from adding the first 10 or so stocks, and that holding 50 stocks in the portfolio gets rid of almost all unsystematic risk.

For this research our objective is to find the optimal point on the line for both Asia and China that exposes an active investor to just enough risk that he has reduced the possibility of

any one company wiping out his portfolio, and enough risk that he is able to outperform the market.

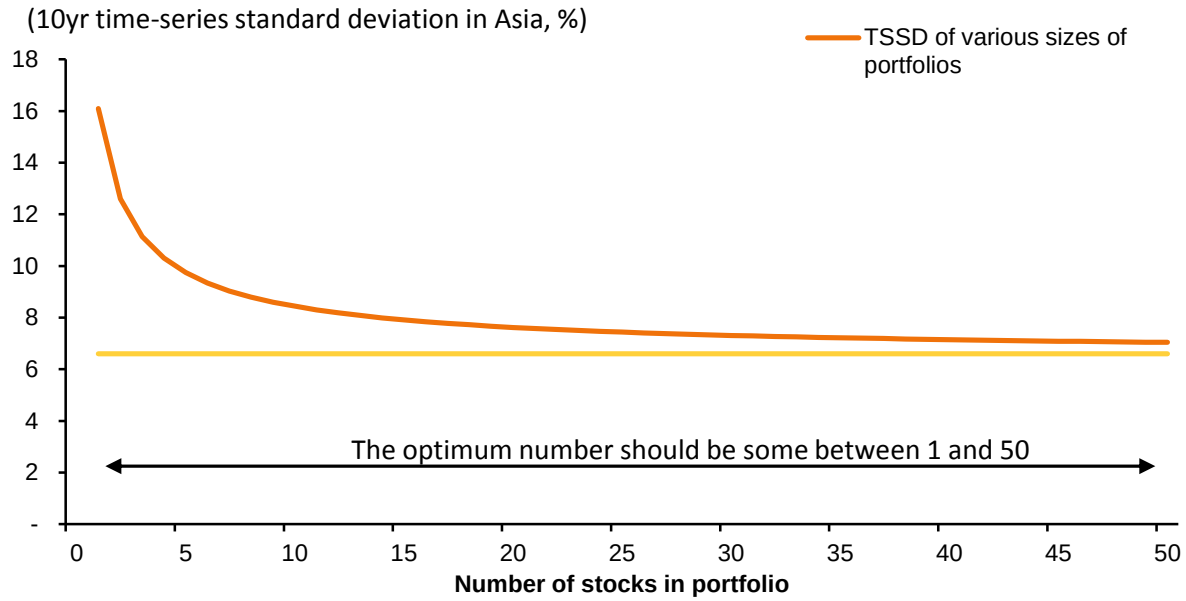

Figure 4. Segmentation of risk in Asia, from one to 50 stocks. To get a better picture of the challenge, rather than carrying the chart to the market portfolio, this chart focuses on portfolio sizes where the optimal number is likely located.

In the Chinese market, the market standard deviation starts quite high, while the single-stock standard deviation is very similar to Asia, which means that the gap between the two is considerably smaller. But, since this these numbers are both derived from squaring the variance, the method for calculating this difference is by taking the differences of the variances and then taking the square root of that. So the gap between single-stock total risk in Asia was 14.7 percentage points while the gap in China was 11.0.

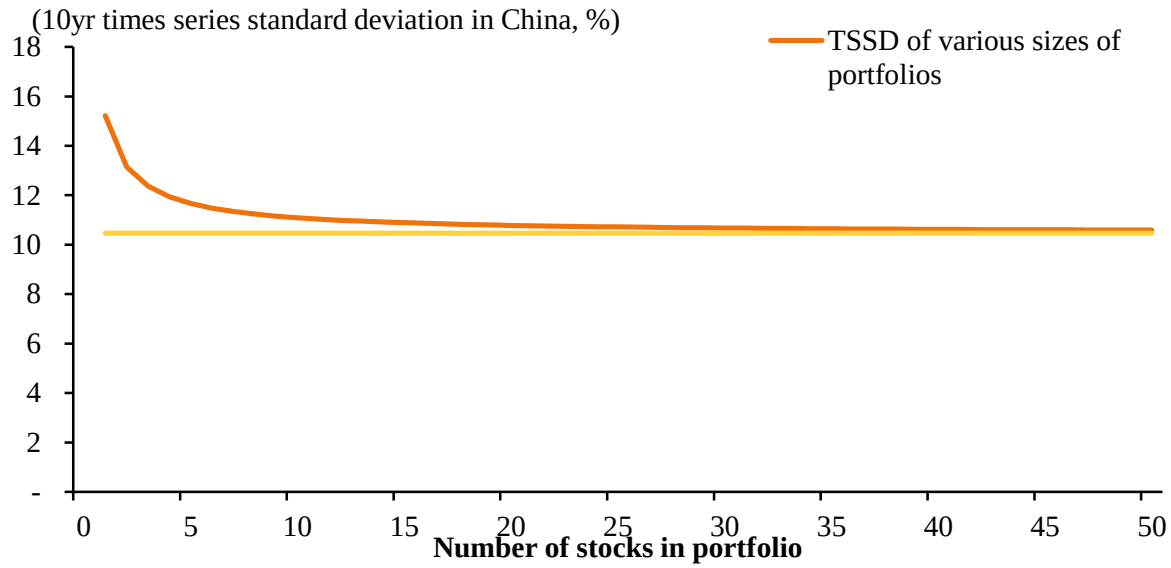

Figure 5. Segmentation of risk in China, from one to 50 stocks. To get a better picture of the challenge, rather than carrying the chart to the market portfolio, this chart focuses on portfolio sizes where the optimal number is likely located.

B. Determining this optimum point

To make this determination, we start by calculating the unsystematic risk, the difference between the lines (described above), which in Asia is 14.7% and in China is 11.0%. Unsystematic risk eventually goes to zero as the portfolio size approaches the total market, because at that point, the impact of any one company going down (or up) is miniscule.

Next, we calculate the percentage of unsystematic risk that is diversified away as we add stocks. For instance, in Asia, by the time an active investor has added the third stock, she has diversified away 39% of the unsystematic risk. With a 15-stock portfolio, that moves to 70% and at 35 stocks it moves to 80%. But to reach 90%, an active investor would need to hold 130 stocks. This demonstrates the exponential impact of the first few stocks.

In China, by the third stock, the investor has diversified away slighty more than 40% of the unsystematic risk, with a 15-stock portfolio, that moves to 73%, at 35 stocks it moves to 82%, but to reach 90%, an active investor would need to hold 101 stocks.

Table VI. Unsystematic risk in Asia and % reduction as stocks are added

	Asia		China	
Stocks in portfolio	Unsystematic risk (%)	% of unsystematic risk removed	Unsystematic risk (%)	% of unsystematic risk removed
1	14.68	-	11.04	-
2	10.73	26.97	7.97	27.91
3	8.97	38.98	6.60	40.35
4	7.92	46.19	5.74	48.09
5	7.18	51.23	5.16	53.37
6	6.61	55.11	4.72	57.39
7	6.16	58.15	4.38	60.44
8	5.81	60.58	4.11	62.95
9	5.51	62.61	3.87	65.09
10	5.26	64.34	3.68	66.80
11	5.03	65.89	3.51	68.34
12	4.84	67.20	3.36	69.70
13	4.66	68.40	3.23	70.85
14	4.50	69.49	3.11	71.95
15	4.36	70.45	3.01	72.87
16	4.23	71.34	2.92	73.71
17	4.11	72.18	2.84	74.47
18	4.00	72.90	2.76	75.20
19	3.90	73.61	2.67	75.93
20	3.81	74.25	2.61	76.55
21	3.72	74.83	2.54	77.14
22	3.64	75.38	2.48	77.65
23	3.57	75.89	2.42	78.22
24	3.49	76.40	2.37	78.69
25	3.43	76.85	2.33	79.10

Figure 6 gives a graphic representation of the portion of unsystematic risk that is removed for each portfolio and shows that Asia and China follow nearly identical trajectories.

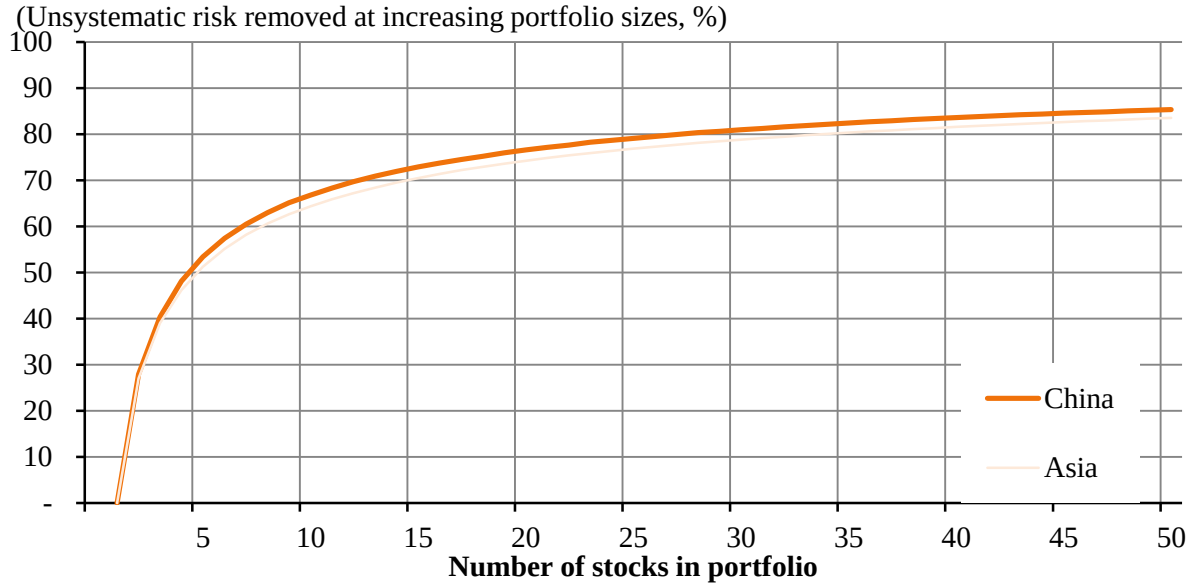

Figure 6. Reduction of unsystematic risk in Asia and China from one to 50 stocks. We shift the focus to the percentage of unsystematic risk removed to help identify at what point an additional stock adds less risk-reduction benefit.

C. *If most unsystematic risk is removed, so is unsystematic return*

At first pass it may seem that an active fund manager should try to diversify away as much risk as possible. But, we must remember that an active investor must bear risk to produce higher return from it. Without bearing that risk, the chance of *deliberately* (as opposed to due to luck) outperforming is reduced.

To understand this risk-return trade-off better, Figure 7 below shows not only risk but also return as stocks are randomly added to a portfolio in Asia. It shows the near-linear fall in return as an investor adds stocks to the portfolio. For example, if an investor added five stocks to his portfolio of Asian stocks he would have eliminated about 51% of unsystematic risk, but only 31% of unsystematic return.

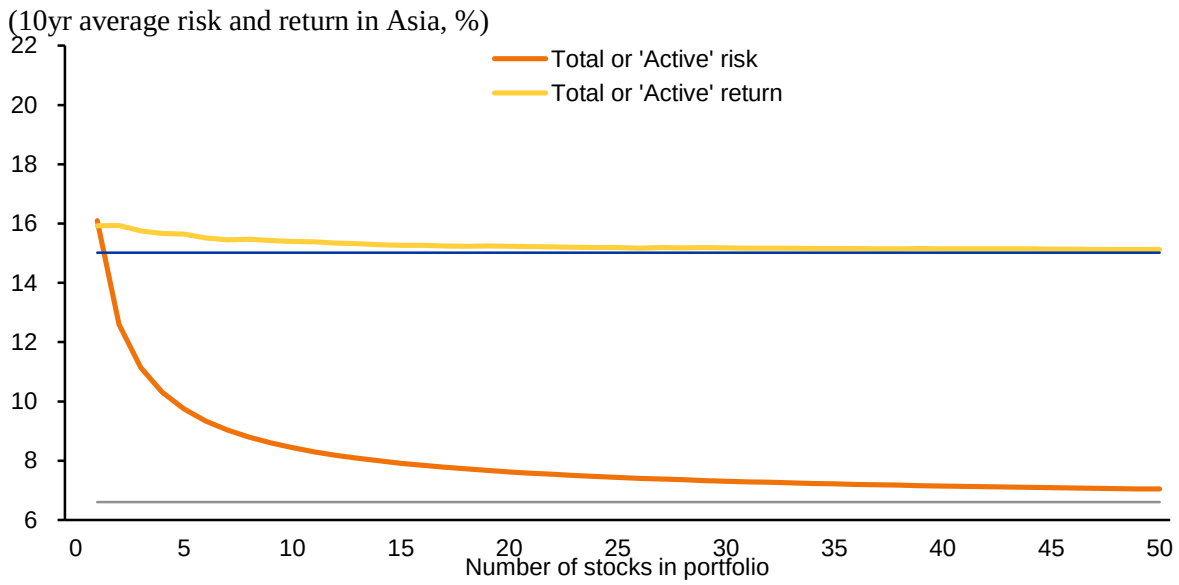

Figure 7. Reduction of risk and return in Asia from one to 50 stocks. The fall in risk in Asia from a single-stock portfolio to the market portfolio is exponential. Portfolios containing a small number of stocks also tend to produce higher returns, which falls to the market average return in more of a linear fashion. The objective of an active fund manager is to balance these opposing factors.

In China, Figure 8 below shows a less-than-linear fall in return as an investor added stocks to the portfolio. From two to seven stocks the fall was exponential, though it moves to a more linear pattern after that. For example, if an investor added five stocks to his portfolio he would have eliminated about 53% of unsystematic risk, similar to that in Asia. But the investor would have removed a whopping 72% of unsystematic return, implying that that the damage of a lower return as the result of diversification was much higher in China than it was in Asia.

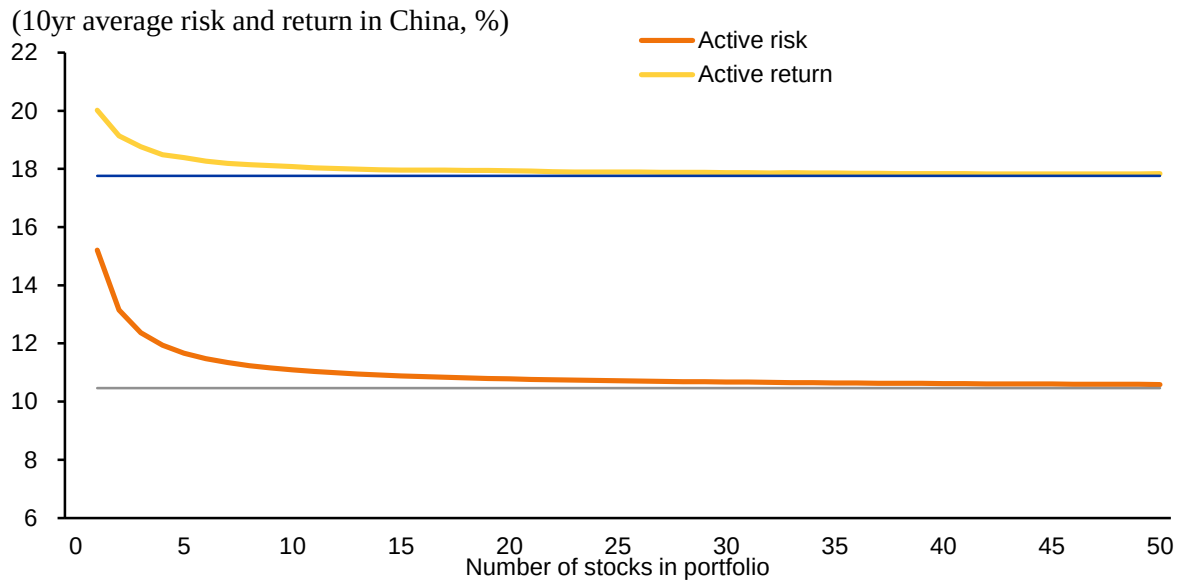

Figure 8. Reduction of risk and return in China from one to 50 stocks. The fall in risk in China from a single-stock portfolio to the market portfolio is exponential. Portfolios containing a small number of stocks also tend to produce higher return, which falls to the market average return in more of a exponential fashion than in Asia. The objective of an active fund manager is to balance these opposing factors, in China this balance tilted towards less stocks in a portfolio.

To further illustrate this dilemma, we calculate the percentage of unsystematic risk and return removed as stocks are added to a randomly selected portfolio in Asia. Before attempting to determine the optimum number, Figures 7 and 8 reminds us that removing too much unsystematic risk can mean removing too much unsystematic return, thus reducing the chance of an active fund manager *deliberately* outperforming.

Furthermore, if an active investor added 25 stocks to their portfolio they would have removed 77% of unsystematic risk, but also 82% of unsystematic return. And, if that was carried out to 50 stocks it would be 84% and 88%, respectively. This illustrates the dilemma that the active investor faces if he holds too many stocks, as he adds stocks – he reduces risk, but also, he quickly reduces his opportunity for *deliberate* outperformance.

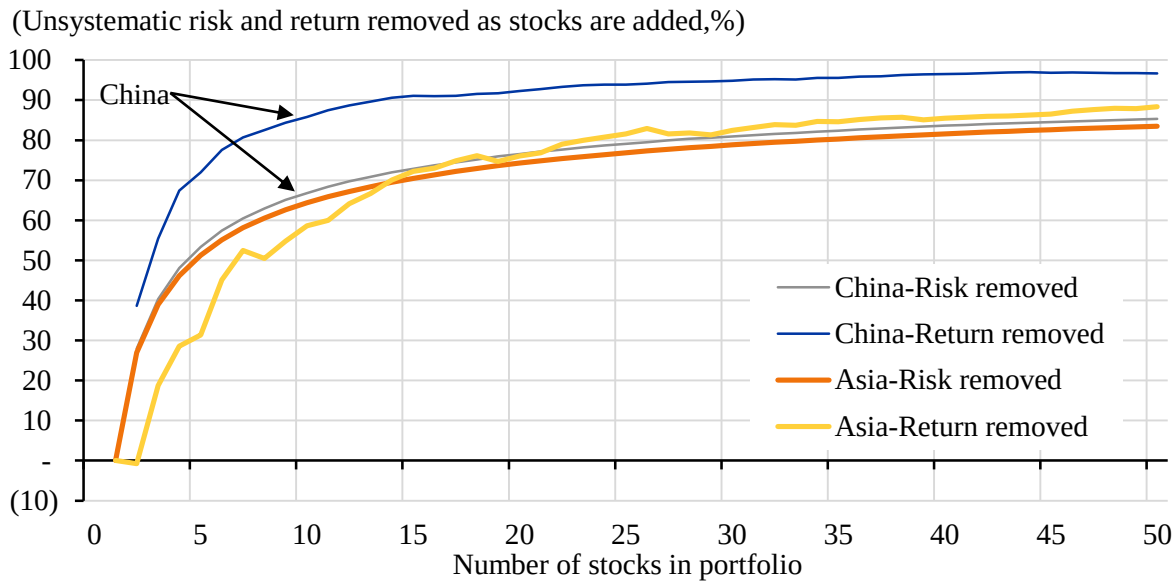

Figure 9. Removal of unsystematic risk and return in Asia and China does not take many stocks. Eventually both unsystematic risk and return is removed when the portfolio contains all stocks in the market. But, most of the impact happens by the time 50 stocks are reached. In Asia, at 50 stocks, 84% of unsystematic risk is removed, while 88% of unsystematic return is removed. In China, at 50 stocks, 85% of unsystematic risk is removed, while 97% of unsystematic return is removed.

D. Equal weight beats naïve weight, but only by a little

Our methodology randomly selects stocks across Asia, or across sectors in the case of China. This naturally tends to give heavier weighting in portfolios to countries and sectors that have the most number of stocks. However, another option would be to equally weight countries in the portfolio. Figure 7 below shows the equally weighted portfolio line starting from 10 stocks as that would mean one stock from each country or from each sector when considered within China..

A 10-stock portfolio using our original data had an average TSSD of 8.4%, which could be reduced to 7.9% by equally weighting. This 60-basis-point reduction seems to be consistent across all portfolio sizes. To us, the slight risk reduction is not worth the added procedure.

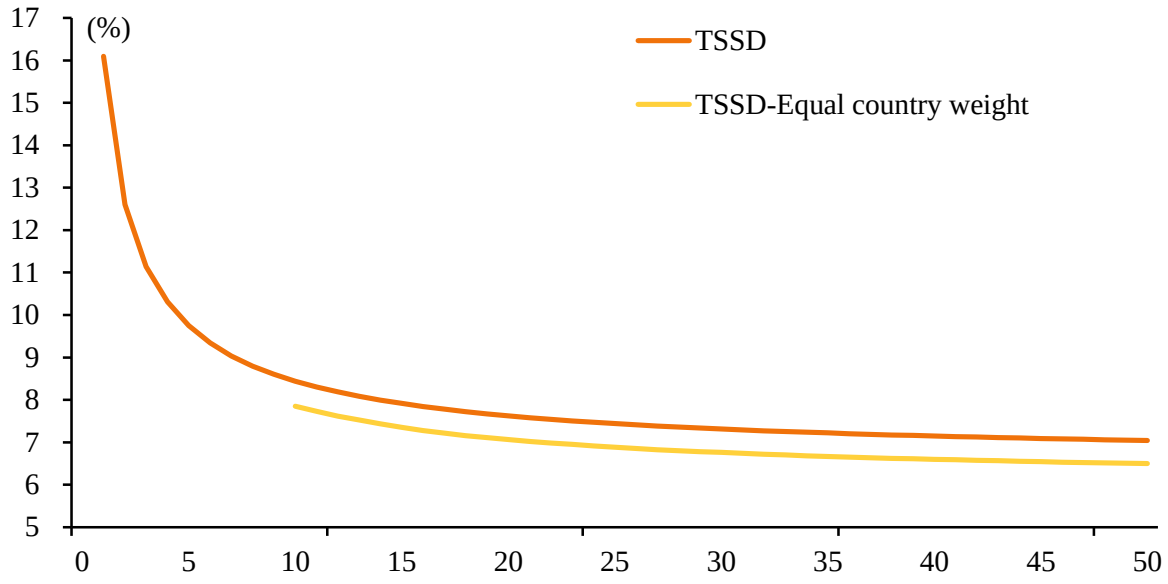

Figure 10. Equal weight gives slightly more risk reduction. We compare our randomly selected portfolio to an equally country weighted portfolio and find a 60-basis-point improvement, though the method requires more procedure, of which the costs may not offset the small benefit.

A 10-stock portfolio of Chinese stocks, using our original data, had an average TSSD of 11.1%, which could be reduced to 10.3% by equally weighting. This 77-basis-point reduction falls to about 67 basis points by the time 38 stocks are in the portfolio. From there, it seems to stay at about 67 basis points onward. So equal sector weighting in China helps more than equal country weigthing in Asia. So for an investor trying to squeeze out every bit of risk reduction, the additional effort of equal weighting may be worth it, but to us, this risk reduction is barely worth the added procedure.

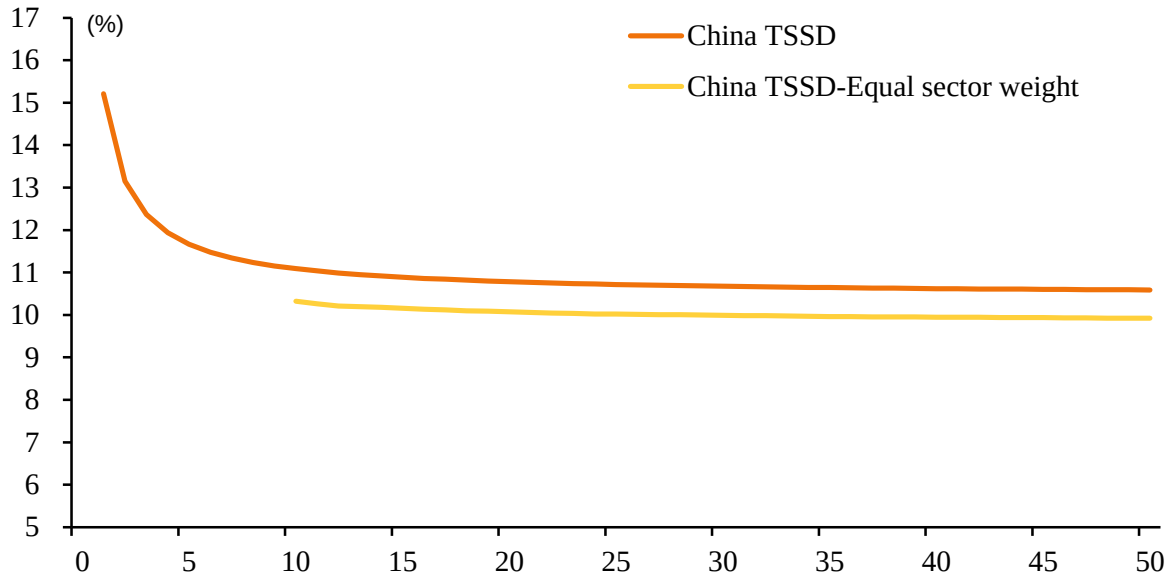

Figure 11. Equal weight gives slightly more risk reduction. We compare our randomly selected portfolio to an equally country weighted portfolio and find a 60-basis-point improvement, though the method requires more procedure, of which the costs may not offset the small benefit.

VI. Analysis

The challenge of determining the optimum number of stocks is applying judgment to select the right point along the unsystematic-risk line, without overly damaging the chance for better-than-market return. We do this through a step-wise process of considering the impact of adding each additional stock to the portfolio.

A. Marginal impact on unsystematic risk reduction

To start, we find that a two-stock portfolio in Asia contains unsystematic risk of 10.73%; this is a 26.97% reduction in unsystematic risk compared to a one-stock portfolio's 14.68%. Then, by adding a third stock in Asia, unsystematic risk falls to 8.97%, a reduction of 16.38%, which is a smaller magnitude compared to moving from a one-stock to a two-stock portfolio. We step through this process to find a point (or range of points) at which the marginal benefit is small enough that the cost of an additional stock is not compensated for.

Though an active investor could be tempted to play it safe and hold as much as 25, 50 or maybe even 100 stocks, this research shows that such a portfolio would be bearing nearly no unsystematic risk and hence would have a very low probability of *deliberate* outperformance. Indeed, in such a case, if outperformance were recorded in any one year or over a series of years, it is likely that such results would be attributed to luck.

Basic statistics tell us that a certain percentage (for example, based on a normal distribution and a one standard deviation, about 15% of outcomes would have to be extreme outperformers, as well as another 15% would be extreme underperformers) of outcomes will be extreme in any one year or over a series of years. This topic of "false discoveries" is thoroughly covered in Barras, Scaillet, Wermers (2005) and in Fama and French (2009).

Table VII. Marginal impact of adding each additional stock in Asia

Stocks in portfolio	Unsystematic risk (%)	% of unsystematic risk removed	% Reduction in unsystematic risk	% Change in reduction
1	14.68	-		
2	10.73	26.97	(26.90)	
3	8.97	38.98	(16.38)	(39.10)
4	7.92	46.19	(11.77)	(28.16)
5	7.18	51.23	(9.33)	(20.74)
6	6.61	55.11	(7.92)	(15.10)
7	6.16	58.15	(6.72)	(15.15)
8	5.81	60.58	(5.77)	(14.09)
9	5.51	62.61	(5.11)	(11.46)
10	5.26	64.34	(4.61)	(9.90)
11	5.03	65.89	(4.33)	(6.05)
12	4.84	67.20	(3.79)	(12.35)
13	4.66	68.40	(3.64)	(4.01)
14	4.50	69.49	(3.41)	(6.20)
15	4.36	70.45	(3.13)	(8.20)
16	4.23	71.34	(2.99)	(4.56)
17	4.11	72.18	(2.88)	(3.74)
18	4.00	72.90	(2.57)	(10.61)
19	3.90	73.61	(2.60)	0.85
20	3.81	74.25	(2.41)	(7.14)
21	3.72	74.83	(2.24)	(6.92)
22	3.64	75.38	(2.15)	(4.08)
23	3.57	75.89	(2.04)	(5.18)
24	3.49	76.40	(2.10)	2.88
25	3.43	76.85	(1.89)	(9.75)

With regards to China, we go through the same process and find that a two-stock portfolio in China contains unsystematic risk of 7.97%; this is a 27.86% reduction in unsystematic risk compared to a one-stock portfolio's 11.04%. Then, by adding a third stock in China, unsystematic risk falls to 6.60%, a reduction of 17.21%, which is a smaller magnitude compared to moving from a one-stock to a two-stock portfolio. We step through this process to find a point (or range of points) at which the marginal benefit is small enough that the cost of an additional stock is not compensated for.

Table VIII. Marginal impact of adding each additional stock in China

Stocks in portfolio	Unsystematic risk (%)	% of unsystematic risk removed	% Reduction in unsystematic risk	% Change in reduction
1	11.04	-		
2	7.97	27.91	(27.86)	
3	6.60	40.35	(17.21)	(38.21)
4	5.74	48.09	(12.93)	(24.88)
5	5.16	53.37	(10.12)	(21.71)
6	4.72	57.39	(8.59)	(15.14)
7	4.38	60.44	(7.11)	(17.23)
8	4.11	62.95	(6.31)	(11.24)
9	3.87	65.09	(5.76)	(8.69)
10	3.68	66.80	(4.85)	(15.85)
11	3.51	68.34	(4.63)	(4.48)
12	3.36	69.70	(4.26)	(8.06)
13	3.23	70.85	(3.79)	(11.08)
14	3.11	71.95	(3.75)	(1.07)
15	3.01	72.87	(3.25)	(13.32)
16	2.92	73.71	(3.08)	(5.20)
17	2.84	74.47	(2.84)	(7.67)
18	2.76	75.20	(2.84)	(0.22)
19	2.67	75.93	(2.94)	3.63
20	2.61	76.55	(2.54)	(13.62)
21	2.54	77.14	(2.50)	(1.35)
22	2.48	77.65	(2.24)	(10.76)
23	2.42	78.22	(2.50)	11.79
24	2.37	78.69	(2.15)	(14.06)
25	2.33	79.10	(1.89)	(12.11)

Figure 12 is a graphic representation of the challenge of identifying the optimum portfolio. We can more clearly see the leveling off of the impact from the reduction in unsystematic risk for a portfolio in Asia. The marginal gain from adding more stocks after about 10 stocks becomes small.

From this information, if the objective was to keep the number of stocks to a minimum the active investor could hold about 10 stocks, which would remove about 64% of unsystematic risk. We can also see that adding an additional 15 stocks would only remove an additional 13% of unsystematic risk. And, at this level of 77% removal of unsystematic risk, the active fund manager is looking more like a passive manager and runs the risk of any outperformance coming only from luck, rather than from true stock-selection skill.

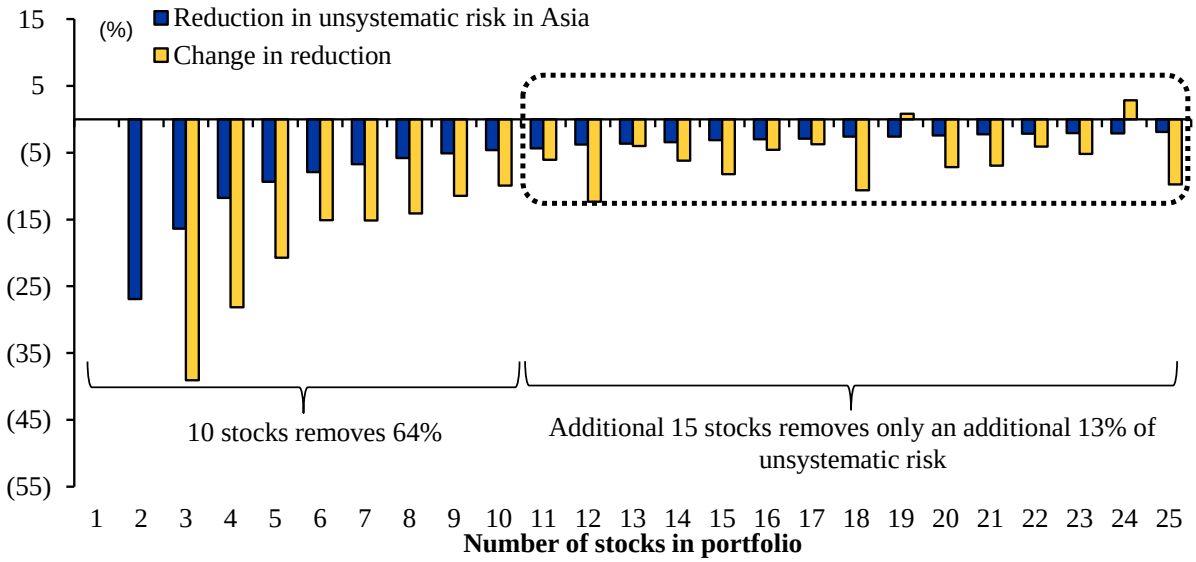

Figure 12. Marginal impact on unsystematic risk from adding each additional stock in Asia. Adding additional stocks to a portfolio must reduce risk, but at some point that reduction levels out, which happens in Asia after about 10 stocks. Though an investor could continue to reduce risk, as she adds stocks to her portfolio it lowers the possibility of outperformance.

For China, if the objective was to keep the number of stocks to a minimum the active investor would hold about 10 stocks, the same as in Asia, which would remove about 67% of unsystematic risk. We can also see that adding an additional 15 stocks would only remove an additional 12% of unsystematic risk. And, at this level of 79% removal of unsystematic risk, the active fund manager is looking more like a passive manager and runs the risk of any outperformance coming only from luck, rather than from true stock-selection skill.

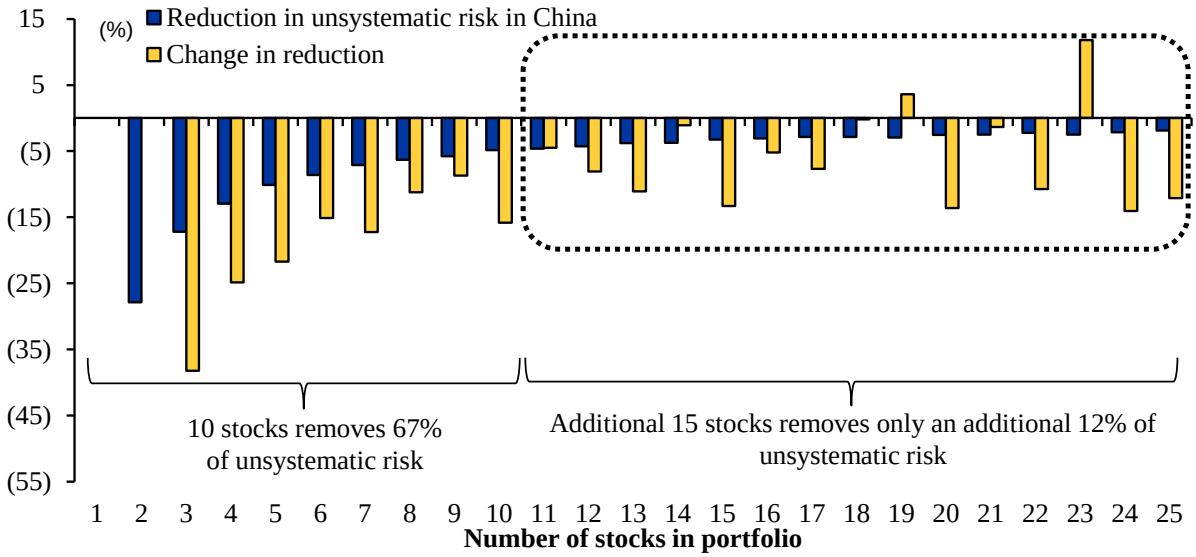

Figure 13. Marginal impact on unsystematic risk from adding each additional stock in China. Adding additional stocks to a portfolio must reduce risk, but at some point that reduction levels out, which happens in China after about 10 stocks, nearly identical to what happens in Asia. Though an investor could continue to reduce risk, as she adds stocks to her portfolio it lowers the possibility of outperformance.

B. Marginal impact on risk-adjusted return reduction

To take Figures 12 and 13 a step further, we divide return by standard deviation to get a rough estimate of a risk-adjusted return. In the case of Asia, we get a risk-adjusted return of 1.0 for the average one-stock portfolio and this improves to 2.3 with a portfolio of all stocks. We take the difference of these two, 1.3 which is the diversifiable portion of risk-adjusted return. We then calculate how much of this would be removed as we add stocks. This helps us understand the loss in return as we add stocks to reduce risk. Considering Asia first, we remove 48% of this by the time we have added five stocks, 65% at 10 stocks and 82% at 25 stocks. Given the more exponential nature of the return line with Chinese stocks, we find that adding each stock can be more damaging to return than is the case with Asia. Hence, we approached the numbers much faster, at five stocks we have removed 68% of diversifiable

risk-adjusted return, 82% at 10 stocks, and 93% by 25 stocks.

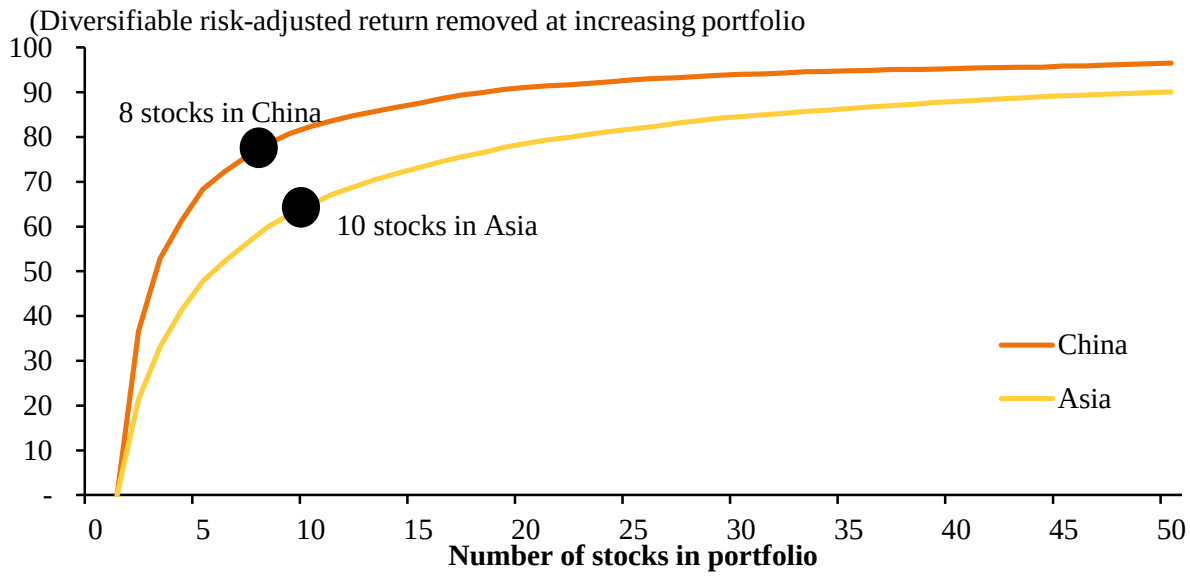

Figure 14. Removal of unsystematic risk and return in Asia does not take many stocks. Eventually, both unsystematic risk and return is removed when the portfolio contains all stocks in the market. But, most of the impact happens by the time 50 stocks are reached. At 50 stocks, 84% of unsystematic risk is removed, while 88% of unsystematic return is removed.

Table IX shows further details of the gains of adding randomly selected stocks to a portfolio in both Asia and China. In our previous paper on the subject (Stotz and Lu 2014), we drew the line at 10 stocks in Asia saying that the benefits of risk reduction and risk-adjusted-return improvement beyond that point where not worth the additional effort and expense, and that additional stocks only would reduce an investor's chance of outperforming the market. We selected this point by identifying the point on the curve where the margin gain from adding an additional stock was slowing. Applying the same guidelines in China, we would draw the line at eight stocks.

Table IX. Unsystematic risk adjusted return in Asia and China and the % reduction as stocks are added

	Asia		China	
Number of stocks	Portion of diversifiable risk adj return removed	% Change	Portion of diversifiable risk adj return removed	% Change
0				
1	-		-	
2	21.4		36.7	
3	33.1	54.4	52.8	43.9
4	41.3	24.8	61.3	16.0
5	47.8	15.8	68.3	11.5
6	52.3	9.3	72.2	5.7
7	56.1	7.3	75.5	4.6
8	59.9	6.7	78.4	3.8
9	62.6	4.5	80.7	2.9
10	64.9	3.7	82.4	2.1
11	67.2	3.5	83.6	1.5
12	68.9	2.5	84.8	1.4
13	70.5	2.4	85.8	1.1
14	71.9	2.0	86.6	1.0
15	73.2	1.8	87.5	1.0
16	74.4	1.8	88.5	1.2
17	75.5	1.5	89.4	0.9
18	76.5	1.3	90.0	0.7
19	77.7	1.6	90.7	0.8
20	78.6	1.1	91.1	0.4
21	79.4	1.0	91.4	0.4
22	80.0	0.8	91.7	0.3
23	80.6	0.8	92.0	0.4
24	81.3	0.8	92.4	0.4
25	81.9	0.7	92.8	0.4

VII. Conclusion

In this research, we identify the optimum number of stocks to own in a portfolio of all stocks in Asia and a portfolio of all Chinese stocks only. In the average randomly selected and equally weighted portfolio, we find that in Asia, 64% of unsystematic risk has been removed by the time a portfolio reaches 10 stocks, and that China at 10 stocks is slightly higher at 67%. If this is carried to 15 stocks then about 70% of unsystematic risk is removed in Asia and about 73% in China. This implies that for portfolios larger than 15 equally weighted stocks, an active fund manager has diversified away most of the unsystematic risk. China stands out as having a higher overall market risk than Asia as a whole and much higher correlations between stocks than any other market in Asia.

The speed of the reduction of portfolio volatility is inversely related to the correlation; when stocks are highly correlated, each new stock added to a portfolio moves much more in synchronization with the portfolio. So an investor needs to add more stocks in a highly correlated market to remove an equal amount of risk. For instance in China, a highly correlated market, an investor needs 19 stocks in a portfolio to reduce the TSSD by 29%, but it takes only a three stock portfolio to get to that point in Asia. This is because China's correlation is 0.46 while Asia's is 0.16.

The return of the average stock in China over this period was 20.01% while the return of all stocks in China was 17.76%, the difference being 2.26%. In Asia, the average stock returned 15.93% versus the market of 15.02%, for a much smaller 0.91% gap. This much wider gap in China versus Asia was closed by the time three stocks were added to the portfolio, which demonstrates that the average return of successive portfolios falls linearly while in China it falls exponentially. Hence, adding stocks helps reduce risk, but it hurts the investor's chance of outperforming the market, and this is amplified in China.

To take into consideration this different behavior in reduction in return, we calculate a risk-adjusted-return measure that can illustrate the balancing act between adding more stocks

to reduce risk and resisting the addition of more stocks to maintain the chance of outperforming. Our conclusion is that at about 10 stocks in Asia the marginal benefit of adding an additional stock starts reducing and this point is reached at about eight stocks in China.

Of course, in real life, an active fund manager faces a large number of constraints which, in most cases, prevents him from having such a concentrated portfolio. However, this research serves as a reminder that from an active-management perspective, a portfolio beyond 8 to 10 stocks runs the risk of producing return and risk profiles similar to the market.

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